

Solutions to Maths workbook – 1 | Trigonometric Identities and Equations

Level - 2	Daily Tutorial Sheet - 11
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131.(D) Squaring and adding given equations we have:

$$25 + 24 \sin(P + Q) = 37 \Rightarrow \sin(P + Q) = \frac{1}{2} \quad \text{So, } P + Q = \frac{\pi}{6} \quad \text{or} \quad \frac{5\pi}{6}$$

$$\text{Case-I: } P + Q = \frac{\pi}{6} \Rightarrow P < \frac{\pi}{6}, Q < \frac{\pi}{6} \Rightarrow 3 \sin P + 4 \cos Q < \frac{3}{2} + 4 < 5.5 < 6$$

$$\therefore P + Q = \frac{\pi}{6} \text{ is not possible} \Rightarrow P + Q = \frac{5\pi}{6} \text{ is only possibility} \Rightarrow R = \frac{\pi}{6}$$

$$\begin{aligned} \mathbf{132.(A)} \quad \frac{V_n - V_{n-1}}{U_{n-1}} &= \frac{\cos n\theta \cdot \sec^n \theta - \cos(n-1)\theta \cdot \sec^{n-1} \theta}{\sin(n-1)\theta \cdot \sec^{n-1} \theta} = \frac{\sec^n \theta [\cos\{(n-1)\theta + \theta\} - \cos(n-1)\theta \cdot \cos \theta]}{\sec^n \theta \cdot \sin(n-1)\theta \cdot \cos \theta} \\ &= \frac{\cos(n-1)\theta \cdot \cos \theta - \sin(n-1)\theta \cdot \sin \theta - \cos(n-1)\theta \cos \theta}{\sin(n-1)\theta \cos \theta} = \frac{-\sin(n-1)\theta \sin \theta}{\sin(n-1)\theta \cos \theta} = -\tan \theta \end{aligned}$$

Alternatively, $V_n = V_{n-1} + U_{n-1} \tan \theta = 0$ because $\cos n\theta = \cos(n-1)\theta \cos \theta - \sin(n-1)\theta \sin \theta$

133.(C) $\sin x + \cos x + \tan x + \cot x + \sec x + \operatorname{cosec} x = 7$

$$\Rightarrow (\sin x + \cos x) + \left(\frac{1}{\sin x \cdot \cos x} \right) + \frac{(\sin x + \cos x)}{\sin x \cdot \cos x} = 7 \Rightarrow (\sin x + \cos x) \left(1 + \frac{2}{\sin 2x} \right) = \left(7 - \frac{2}{\sin 2x} \right)$$

$$\text{Squaring, we have } (1 + \sin 2x) \left(1 + \frac{4}{\sin^2 2x} + \frac{4}{\sin 2x} \right) = 49 + \frac{4}{\sin^2 2x} - \frac{28}{\sin 2x}$$

$$\Rightarrow 1 + \frac{4}{\sin^2 2x} + \frac{4}{\sin 2x} + \sin 2x + \frac{4}{\sin 2x} + 4 = 49 + \frac{4}{\sin^2 2x} - \frac{28}{\sin 2x} \Rightarrow \sin 2x - 44 + \frac{36}{\sin 2x} = 0$$

$$\Rightarrow \sin^2 2x - 44 \sin 2x + 36 = 0 \Rightarrow \sin 2x = \frac{44 \pm \sqrt{1936 - 144}}{2} = \frac{44 \pm \sqrt{1792}}{2} = 22 \pm 8\sqrt{7}$$

$$\sin 2x \leq 1 \Rightarrow \sin 2x = 22 - 8\sqrt{7}$$

$$\therefore a = 22, b = 8 \quad (22 - 8\sqrt{7} = 22 - \sqrt{448} < 1 \text{ because } 22 = \sqrt{484} > \sqrt{448} > \sqrt{441} = 21)$$

134.(A) $\sin A \sin 2A \sin 3A \sin 4A = \sin A (2 \sin A \cos A) (3 \sin A - 4 \sin^3 A) \times 2 \sin 2A \cos 2A$

$$= 2 \sin^2 A \cos A \times \sin A (3 - 4 \sin^2 A) \times 2 \times 2 \sin A \cos A (1 - 2 \sin^2 A)$$

$$= 8 \sin^4 A \cos^2 A (3 - 4 \sin^2 A) (1 - 2 \sin^2 A) = 8x^2 (1-x) (3-4x) (1-2x) = 24x^2 - 104x^3 + 144x^4 - 64x^5$$

The required sum = $24 - 104 + 144 - 64 = 0$

Alternatively, Sum of coefficients can also be obtained by putting $x = 1$ directly in $8x^2 (1-x) (3-4x) (1-2x)$

$$\begin{aligned} \mathbf{135.(D)} \quad \frac{1}{\cos \alpha + \cos(n+1)\alpha} &= \frac{1}{2 \cos(n+1)\alpha \cdot \cos n\alpha} = \frac{\sin \alpha}{2 \sin \alpha \cdot \cos(n+1)\alpha \cdot \cos n\alpha} \\ &= \frac{\sin[(n+1)\alpha - n\alpha]}{2 \sin \alpha \cdot \cos(n+1)\alpha \cdot \cos n\alpha} = \frac{\sin(n+1)\alpha \cdot \cos n\alpha - \cos(n+1)\alpha \cdot \sin n\alpha}{2 \sin \alpha \cdot \cos(n+1)\alpha \cdot \cos n\alpha} \end{aligned}$$

$$= \frac{1}{2} \operatorname{cosec} \alpha \left[\frac{\sin(n+1)\alpha \cdot \cos n\alpha}{\cos(n+1)\alpha \cdot \cos n\alpha} - \frac{\cos(n+1)\alpha \cdot \sin n\alpha}{\cos(n+1)\alpha \cdot \cos n\alpha} \right] = \frac{1}{2} \operatorname{cosec} \alpha [\tan(n+1)\alpha - \tan n\alpha]$$

$$\therefore \frac{1}{\cos \alpha + \cos(2n+1)\alpha} = \frac{1}{2} \operatorname{cosec} \alpha [\tan(n+1)\alpha - \tan n\alpha]$$

Put $n = 1, 2, \dots$. Then $\frac{1}{\cos \alpha + \cos 3\alpha} = \frac{1}{2} \operatorname{cosec} \alpha [\tan 2\alpha - \tan \alpha]$.

$$\frac{1}{\cos \alpha + \cos 5\alpha} = \frac{1}{2} \operatorname{cosec} \alpha [\tan 3\alpha - \tan 2\alpha] \dots$$

$$\frac{1}{\cos \alpha + \cos 3\alpha} + \frac{1}{\cos \alpha + \cos 5\alpha} + \dots + \frac{1}{\cos \alpha + \cos(2n+1)\alpha}$$

$$= \frac{1}{2} \operatorname{cosec} \alpha [(\tan 2\alpha - \tan \alpha) + (\tan 3\alpha - \tan 2\alpha) + \dots + \{ \tan(n+1)\alpha - \tan n\alpha \}] = \frac{1}{2} \operatorname{cosec} \alpha [\tan(n+1)\alpha - \tan \alpha]$$

136.(A) $-\sqrt{2} \leq \sin x - \cos x \leq \sqrt{2} \Rightarrow -2 \leq \sqrt{2}(\sin x - \cos x) \leq 2 \Rightarrow 1 \leq \sqrt{2}(\sin x - \cos x) + 3 \leq 5$

$$\Rightarrow 0 \leq \log_{\sqrt{5}} [\sqrt{2}(\sin x - \cos x) + 3] \leq 2$$

137.(ABD) $3^{\sin 2x + 2\cos^2 x} + 3^{1 - \sin 2x + 2\sin^2 x} = 28 \Rightarrow 3^{\sin 2x + 2\cos^2 x} + 3^{3 - (\sin 2x + 2\cos^2 x)} = 28$

Put $3^{\sin 2x + 2\cos^2 x} = y \Rightarrow y + \frac{27}{y} = 28 \Rightarrow y^2 - 28y + 27 = 0 \Rightarrow (y-1)(y-27) = 0 \Rightarrow y = 1 \text{ or } 27$

$$\therefore 3^{\sin 2x + 2\cos^2 x} = 1 \text{ or } 3^3 \Rightarrow \sin 2x + 2\cos^2 x = 0 \text{ or } 3$$

$\sin 2x + 2\cos^2 x = 3$ isn't possible

$$\sin 2x + 2\cos^2 x = 1 + \sin 2x + \cos 2x \leq \sqrt{2} + 1 < 3$$

$$\Rightarrow 2\cos x(\sin x + \cos x) = 0$$

$$\Rightarrow \cos x = 0 \text{ or } \tan x = -1. \text{ Also } \sin x = -1 \Rightarrow \cos x = 0$$

138.(A) $4^{\sin 2x + 2\cos^2 x} + 4^{1 - \sin 2x + 2\sin^2 x} = 65 \Rightarrow 4^{\sin 2x + \cos 2x + 1} + 4^{1 - \sin 2x - \cos 2x + 1} = 65$

$$4^{\sin 2x + \cos 2x + 1} + 4^{3 - (\sin 2x + \cos 2x)} = 65,$$

Let $4^{\sin 2x + \cos 2x + 1} = y$

$$y + \frac{64}{y} = 65 \Rightarrow y^2 - 65y + 64 = 0 \Rightarrow y = 1, y = 64 \Rightarrow 4^{\sin 2x + \cos 2x + 1} = 4^0 \text{ or } 4^{\sin 2x + \cos 2x + 1} = 4^3$$

$$1 + \sin 2x + \cos 2x \leq \sqrt{2} + 1 < 3$$

$$\Rightarrow \sin 2x + \cos 2x = -1$$

139.(C) $\sin^{10} x - \cos^{10} x = 1 \Rightarrow \sin^{10} x = 1 + \cos^{10} x$

$$\sin^{10} x \leq 1 \Rightarrow 1 + \cos^{10} x \leq 1 \Rightarrow \cos^{10} x = 0 \Rightarrow \cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}$$

140.(B) $\sin \left[\left(\frac{\pi}{4} \right) \cot \theta \right] = \cos \left[\left(\frac{\pi}{4} \right) \tan \theta \right] \Rightarrow \cos \left[\left(\frac{\pi}{2} \right) - \left(\frac{\pi}{4} \right) \cot \theta \right] = \cos \left[\left(\frac{\pi}{4} \right) \tan \theta \right]$

$$\Rightarrow \frac{\pi}{2} - \frac{\pi}{4} \cot \theta = 2m\pi \pm \frac{\pi}{4} \tan \theta \Rightarrow \frac{\pi}{4} (\tan \theta - \cot \theta) = \left(2m - \frac{1}{2} \right) \pi \text{ or } \frac{\pi}{4} (\cot \theta + \tan \theta) = \frac{\pi}{2} (1 - 4m)$$

$$\Rightarrow \tan \theta - \cot \theta = 2(4m - 1) \text{ or } \tan \theta + \cot \theta = 2(1 - 4m), m \in \mathbb{Z}$$

This is the complete solution. Then for $m = 0$, we get $\tan \theta + \cot \theta = 2$

$$\Rightarrow 1 + \tan^2 \theta = 2 \tan \theta$$

$$\Rightarrow \tan^2 \theta - 2 \tan \theta + 1 = 0 \Rightarrow (\tan \theta - 1)^2 = 0 \Rightarrow \tan \theta = 1 = \tan \left(\frac{\pi}{4} \right) \Rightarrow \theta = n\pi + \left(\frac{\pi}{4} \right), n \in \mathbb{Z}$$

All other options won't satisfy the complete solution.